E

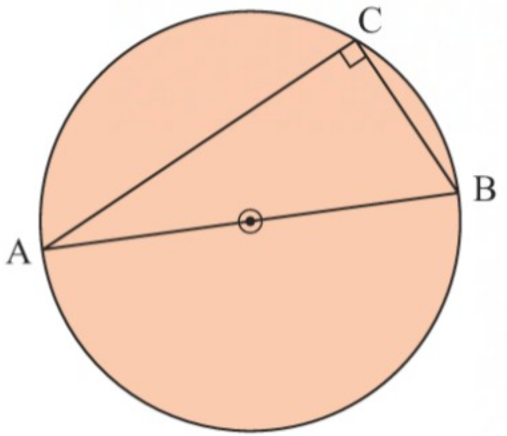
CIRCLE PROBLEMS

There are also certain properties of circles which involve right angles. They are described in the following theorems:

ANGLE IN A SEMI-CIRCLE

> The angle in a semi-circle is a right angle.

No matter where C is placed on the circle, $A\widehat{C}B$ is always a right angle.



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Example 11 **Self Tutor**

A circle has diameter [XY] of length 13 cm. Z is a point on the circle such that XZ is 5 cm. Find the length YZ.

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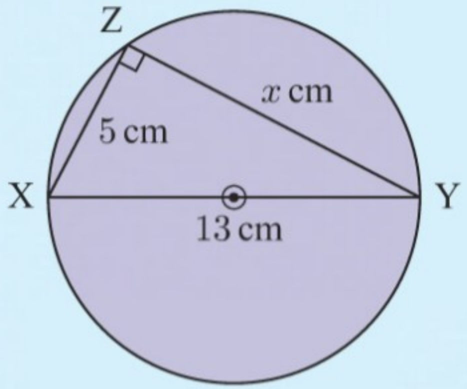
From the angle in a semi-circle theorem, $X\widehat{Z}Y$ is a right angle.

Let the length YZ be x cm.

$$\begin{aligned} \therefore\ 5^2 + x^2 &= 13^2 && \{\text{Pythagoras}\} \\ \therefore\ x^2 &= 169 - 25 = 144 \\ \therefore\ x &= \sqrt{144} && \{\text{as } x > 0\} \\ \therefore\ x &= 12 \end{aligned}$$

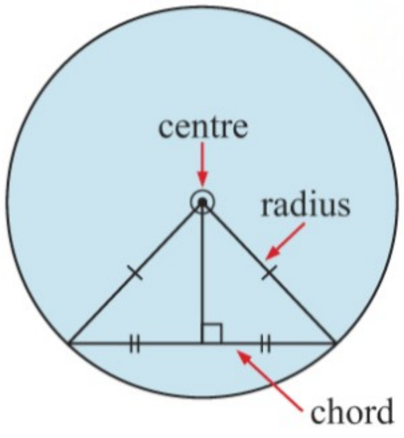
So, YZ has length 12 cm.



A CHORD OF A CIRCLE

The line drawn from the centre of a circle at right angles to a chord, bisects the chord.

The construction of radii from the centre of the circle to the end points of the chord produces an isosceles triangle. The above property then follows from the **isosceles triangle theorem**.



Example 12 **Self Tutor**

A circle with radius 8 cm has a chord of length 10 cm.
Find the shortest distance from the centre of the circle to the chord.

Example 12

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Find the shortest distance from the centre of the circle to the chord.

The shortest distance is the 'perpendicular distance'. The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.

$\therefore$ AB = BC = 5 cm

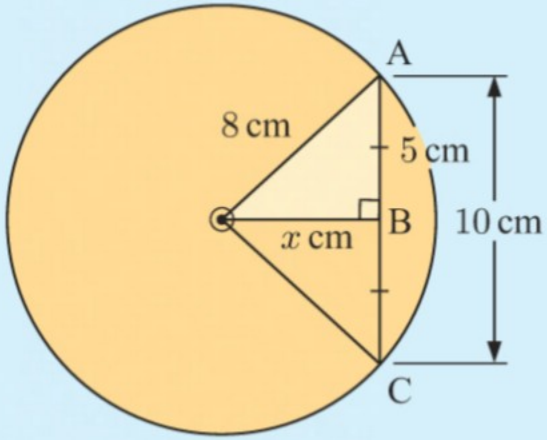
In $\triangle$AOB, $5^2 + x^2 = 8^2$ {Pythagoras}

$\therefore x^2 = 64 - 25 = 39$

$\therefore x = \sqrt{39}$ {as $x > 0$}

$\therefore x \approx 6.24$

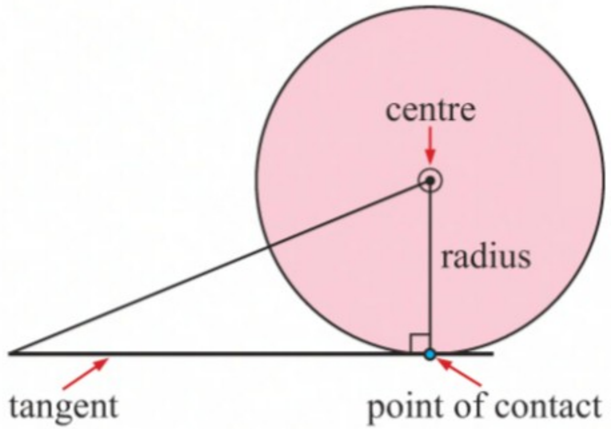
So, the shortest distance is about 6.24 cm.



TANGENT-RADIUS PROPERTY

A tangent to a circle and a radius at the point of contact meet at right angles.

Notice that we can now form a right angled triangle.



Example 13 **Self Tutor**

A tangent of length 10 cm is drawn to a circle with radius 7 cm. How far is the centre of the circle from the end point of the tangent?

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Let the distance be d cm.

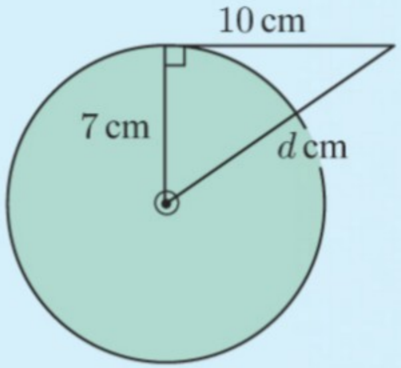
$\therefore\ d^2 = 7^2 + 10^2$ {Pythagoras}

$\therefore\ d^2 = 149$

$\therefore\ d = \sqrt{149}$ {as $d > 0$}

$\therefore\ d \approx 12.2$

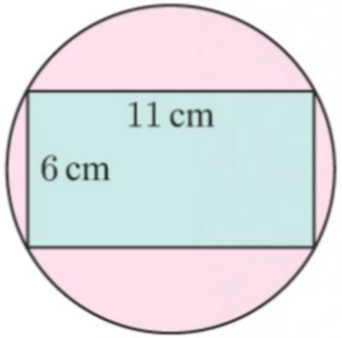
The centre is about 12.2 cm from the end point of the tangent.



EXERCISE 5E

1 A circle has diameter [AB] of length 10 cm. C is a point on the circle such that AC is 8 cm. Find the length BC.

2 A rectangle with side lengths 11 cm and 6 cm is inscribed in a circle. Find the radius of the circle.



3 A circle with radius 4 cm has a chord of length 3 cm. Find the shortest distance from the centre of the circle to the chord.

4 A chord of length 6 cm is 3 cm from the centre of a circle. Find the radius of the circle.

5 A chord is 5 cm from the centre of a circle of radius 8 cm. Find the length of the chord.

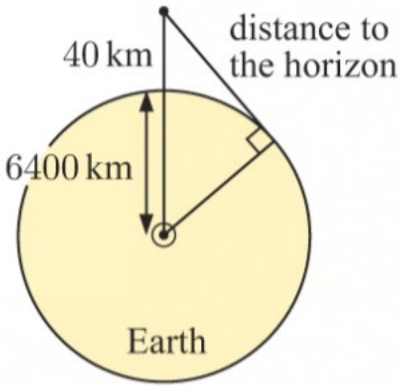
6 A circle has radius 3 cm. A tangent is drawn to the circle from point P, which is 9 cm from the circle's centre. How long is the tangent?

7 A tangent of length 12 cm has end point 16 cm from the circle's centre. Find the radius of the circle.

8 A circular table of diameter 2 m is placed in the corner of a room so that its edges touch two perpendicular walls. Find the shortest distance from the corner of the room to the edge of the table.

9 The radius of the Earth is about 6400 km.

Determine the distance to the horizon from a rocket which is 40 km above the Earth's surface.



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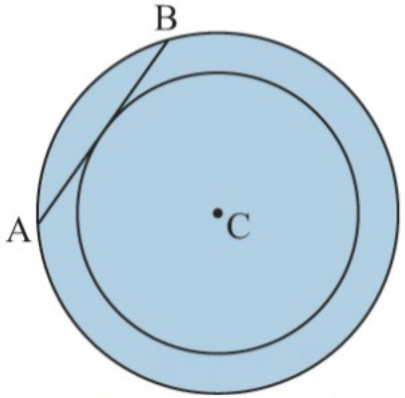
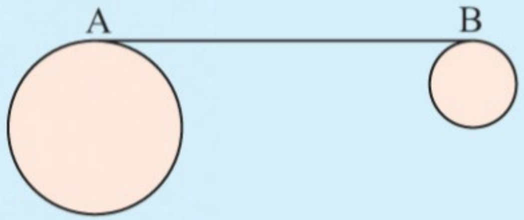


C is the centre of two circles with radii 7 cm and 5 cm. [AB] is a chord of the larger circle, and a tangent of the smaller circle.

Find the length of [AB].

Example 14 **Self Tutor**

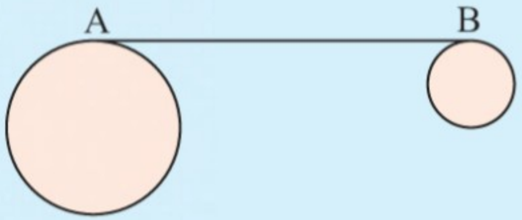
Two circles have a common tangent with points of contact A and B which are 7 cm apart. The radii of the circles are 4 cm and 2 cm respectively. Find the distance between the centres.

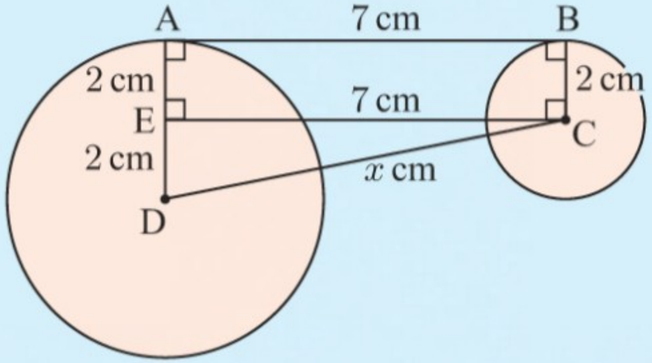


Example 14

Self Tutor

Two circles have a common tangent with points of contact A and B which are 7 cm apart. The radii of the circles are 4 cm and 2 cm respectively. Find the distance between the centres.





For centres C and D, we draw [BC], [AD], [CD], and [CE] $\parallel$ [AB].

$\therefore$ ABCE is a rectangle.

$\therefore$ CE $= 7$ cm {as CE $=$ AB}

and DE $= 4 - 2 = 2$ cm

Let the distance between the centres be x cm.

$\therefore\ x^2 = 2^2 + 7^2$ {Pythagoras in $\triangle$DEC}

$\therefore\ x^2 = 53$

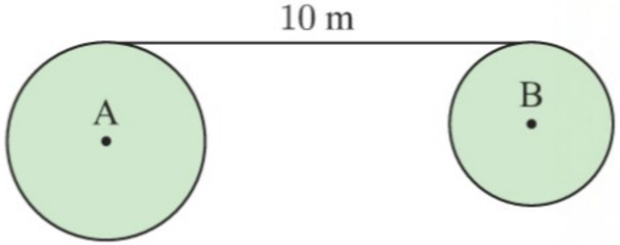
$\therefore\ x = \sqrt{53}$ {as $x > 0$}

$\therefore\ x \approx 7.28$

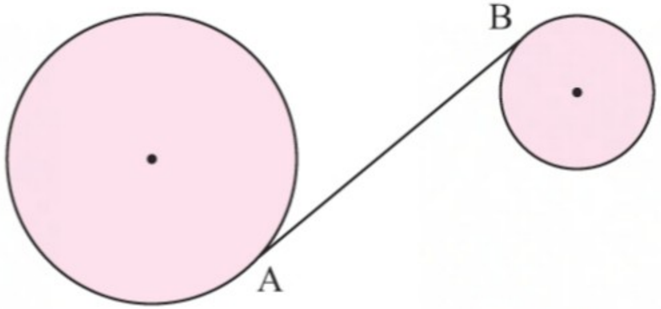
The distance between the centres is about 7.28 cm.

11 A and B are the centres of two circles with radii 4 m and 3 m respectively. The illustrated common tangent has length 10 m.

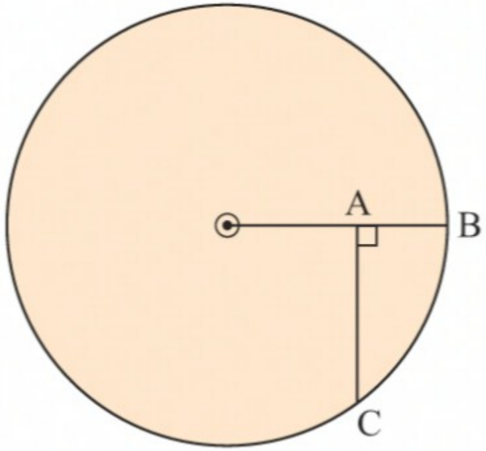
Find the distance between the centres to the nearest millimetre.



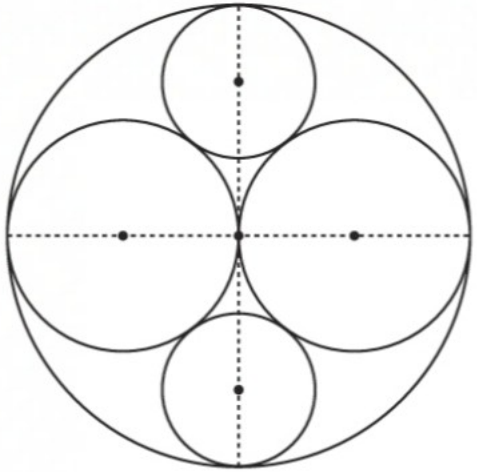
12 The illustration shows two circles of radii 4 cm and 2 cm respectively. The distance between the two centres is 8 cm. Find the length of the common tangent [AB].



13 In the given figure, AB = 1 cm and AC = 3 cm. Find the radius of the circle.



14 In the given figure, the largest circle has radius 12 cm.



a Find the radius of:

i the medium circles

ii the smallest circles.

b What fraction of the largest circle is occupied by the four inner circles?

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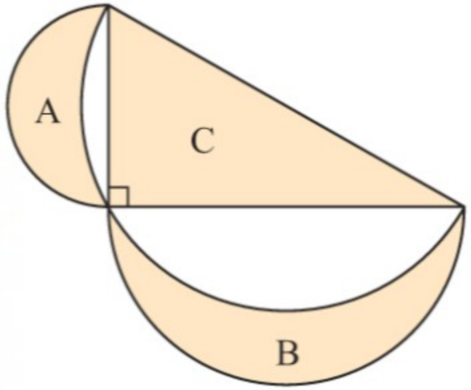

Show that Area A + Area B = Area C.

EXERCISE 5E

1 BC $= 6$ cm **2** ≈ 6.26 cm **3** ≈ 3.71 cm

4 $3\sqrt{2}$ cm ≈ 4.24 cm **5** $2\sqrt{39}$ cm ≈ 12.5 cm

6 $6\sqrt{2}$ cm ≈ 8.49 cm **7** $4\sqrt{7}$ cm ≈ 10.6 cm

8 $(\sqrt{2}-1)$ m ≈ 41.4 cm **9** ≈ 717 km

10 AB $= 4\sqrt{6}$ cm ≈ 9.80 cm **11** $\sqrt{101}$ m ≈ 10.050 m

12 AB $= 2\sqrt{7}$ cm ≈ 5.29 cm **13** 5 cm

14 **a** **i** 6 cm **ii** 4 cm **b** $\frac{13}{18}$

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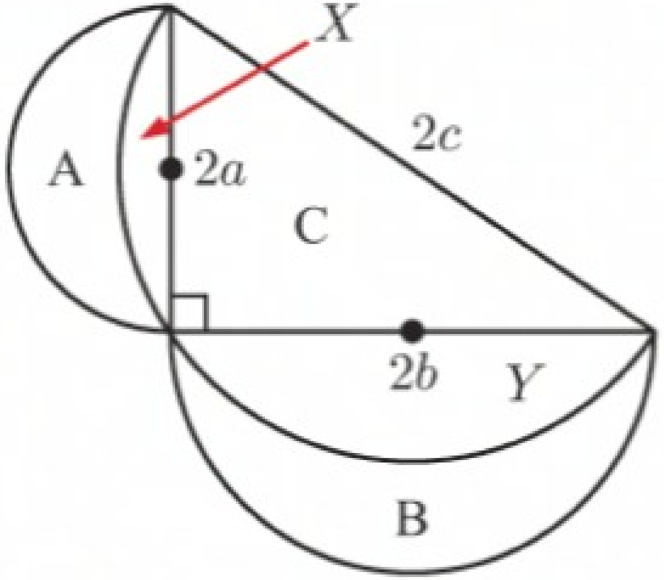

Hint:

$$A + X = \tfrac{1}{2}\pi a^2$$

$$B + Y = \tfrac{1}{2}\pi b^2$$

$$C + X + Y = \tfrac{1}{2}\pi c^2$$